The Voltage Transfer Curve and Stability Criteria in the Theory of the AC Plasma Display

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Abstract—At an ac plasma display discharge site the voltage transfer curve and the locus of the equilibrium points establish conditions which govern the change of wall voltage toward or away from equilibrium levels. Analysis based on simple geometric ideas leads to a precise description of these conditions, and in the neighborhood of the equilibrium levels, the conditions reduce to known results.

I. Introduction

A SEQUENCE of pulsed gas discharges at an ac plasma display discharge site has some properties that are similar to those of a sampled-data regulatory system. In equilibrium, although the wall voltage alternates its sign with every discharge, it preserves its magnitude. If the equilibrium is stable any perturbation in wall voltage damps out in the following discharges; if the equilibrium is unstable a perturbation grows [1].

The behavior of the discharge sequence near equilibrium has been discussed in terms of the voltage transfer curve, which relates change in magnitude of wall voltage during a discharge to the magnitude of the voltage across the discharge site at the beginning of the discharge [1]. Despite the importance of the voltage transfer curve and the locus of equilibrium points in the theory of the ac plasma display, only the behavior of the discharge sequences near equilibrium has been discussed precisely [1]. In the region away from equilibrium, discussion has been only qualitative [2], [3]. The analysis presented here is valid in all regions, and it reduces to the known results at the equilibrium points.

Fig. 1 shows the essential process for a plasma display discharge site in which a gas discharge, initiated by a sufficiently high voltage pulse, reaches equilibrium after a number of cycles. Later, an erase pulse either terminates the sequence abruptly or establishes conditions under which the sequence will terminate itself. The behavior of these discharge sequences is governed by the voltage transfer curve and its relation to the locus of equilibrium points shown superimposed in Fig. 2, which relates the change in wall voltage to the total voltage across the discharge site just before the discharge starts. With the assumption that the discharges in each half cycle are identical except for sign, the magnitude of the change of the wall voltage is then twice the wall voltage itself, and the locus of equilibrium points is a straight line that begins at the applied voltage V_s and extends upward with a slope equal to 2. The intersections A, B, and C of the two curves are the actual equilibrium points.

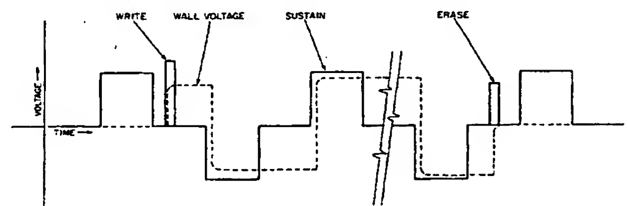

Fig. 1. Initiation, maintenance, and termination of discharge sequence in the ac plasma display.

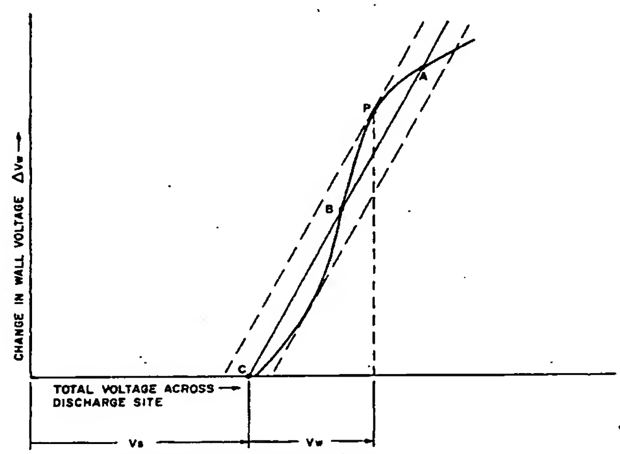

Fig. 2. Voltage transfer curve (curved line intersecting A, B, C, and P), and locus of equilibrium points (straight solid line intersecting A, B, and C). The dashed lines define the limit of bistability.

The dashed lines that are tangent to the voltage transfer curve define the range of bistability for the sustaining voltage V_s. The point P represents a transient operating point with a wall voltage V_w. The ordinate of P represents the change in wall voltage ΔV_w that results from the discharge.

Analysis of the properties of the voltage transfer curve has disclosed that equilibrium discharge sequences will be

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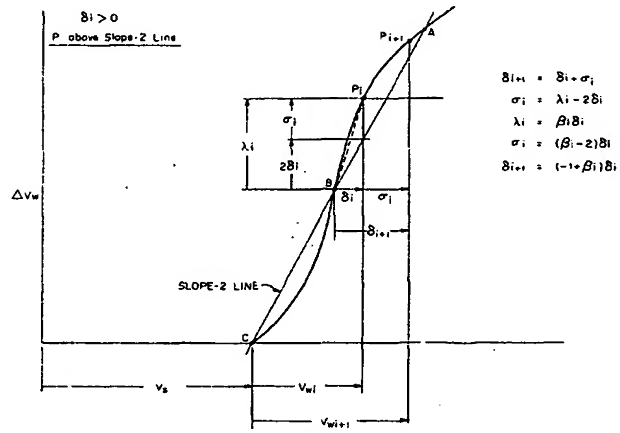

Fig. 3. Movement of operating points in the ac plasma display. Wall voltage greater than equilibrium value. Operating point above locus of equilibrium points ($\beta > 2$).

stable if the slope of the voltage transfer curve is equal to or less than 2 at equilibrium [1]. The defining equation is

$$\delta_{i+1} = (-1 + \gamma)\delta_i \tag{1}$$

where δ_i is the perturbation of wall voltage from equilibrium after the ith discharge, δ_{i+1} is the perturbation after the following discharge, and γ is the slope of the voltage transfer curve at the equilibrium point. In regions away from the neighborhood of equilibrium, (1) is not meaningful. In fact, between adjacent equilibrium points the slope will vary from γ less than 2 to γ greater than 2.

II. Analysis

In Fig. 3, we consider a point P_i to the right of B with a wall voltage V_{wi} that differs from the equilibrium wall voltage at B by δ_i volts. The change in wall voltage required to preserve the same magnitude after the discharge is $2V_{wi}$, but Fig. 3 shows that the actual voltage change will exceed this by σ_i volts. Therefore, the new wall voltage will differ from the equilibrium value at B by

$$\delta_{i+1} = \delta_i + \sigma_i \tag{2}$$

where δ_i is measured from the equilibrium wall voltage and σ_i is measured from the locus of equilibrium points to the voltage transfer curve. For the case illustrated, both δ_i and σ_i are positive, $\delta_{i+1} > \delta_i$, and the next operating point P_{i+1} will be to the right of P_i by σ_i volts. To eliminate σ_i from (2), we first note from Fig. 3 that

$$\sigma_i = \lambda_i - 2\delta_i. \tag{3}$$

Furthermore, λ_i is related to δ_i through the slope β_i of the chord connecting P_i to B,

$$\lambda_i = \beta_i\delta_i. \tag{4}$$

Substitution of (4) into (3) leads to

$$\sigma_i = (\beta_i - 2)\delta_i \tag{5}$$

and substitution of (5) into (2) leads to the expression

$$\delta_{i+1} = (-1 + \beta_i)\delta_i. \tag{6}$$

For this case $\beta_i > 2$ and $\delta_{i+1} > \delta_i$ and P moves away from the equilibrium point at B and toward A.

Fig. 4 illustrates the case for which B is again the reference point, but δ_i, σ_i, and λ_i are negative. The change in wall voltage during the ith discharge is less than that necessary to preserve the magnitude of V_{wi} by an amount $|\sigma_i|$ and P_{i+1} will be to the left of P_i by the same amount and to the left of B by $|\delta_i + \sigma_i|$ volts. This is stated more precisely by (2) which is still valid, and in which δ_{i+1}, δ_i, and σ_i are all negative. The balance of the analysis and (6), to which it leads, are also valid in this case. Since in both the cases considered $\beta > 2$, (6) implies that an operating point different from B will move further away from B at the next discharge. B, then, is a point of unstable equilibrium.

When we shift the reference to the operating point A we only change magnitudes and signs. The algebra, summarized in Figs. 5 and 6, remains the same. In Fig. 5, as in the case just considered, the change in wall voltage is too small to preserve the magnitude of the wall voltage at P_i and the next operating point P_{i+1} is at the left of P_i. In this case $0 < \beta < 1$ and (6) implies that P moves from P_i at the right of A to P_{i+1} at the left of A and that P_{i+1} will be closer to A than P_i.

Equations (2)–(6) are valid for all points on the transfer curve, and (6) states that the difference between a wall voltage for a point P and an adjacent equilibrium wall voltage will diminish if the slope of the chord connecting P to the equilibrium point is between 0 and 2, $0 < \beta < 2$. If $1 < \beta < 2$ for all points closer to equilibrium than P_i, the wall voltage approaches the equilibrium wall voltage either from one side or the other. If $0 < \beta < 1$, for all points closer to equilibrium than P_i, the wall voltage oscillates around the equilibrium wall voltage as the difference between these voltages diminishes with each discharge. If $\beta = 1$, the wall voltage reaches equilibrium in a single discharge. In the limit when P approaches the equilibrium point, the chord approaches the tangent of the voltage transfer curve, (6) becomes identical with (1), and the condition for which perturbations from equilibrium will damp out is

$$0 < \gamma < 2 \tag{7}$$

which agrees with [1, eq. 13].

III. Discussion

According to (6), all operating points above A will move downward toward A; all points between A and B will move upward toward A and away from B; and all points between B and C will move downward away from B toward C (the "off" state). A and C are the stable equilibrium points

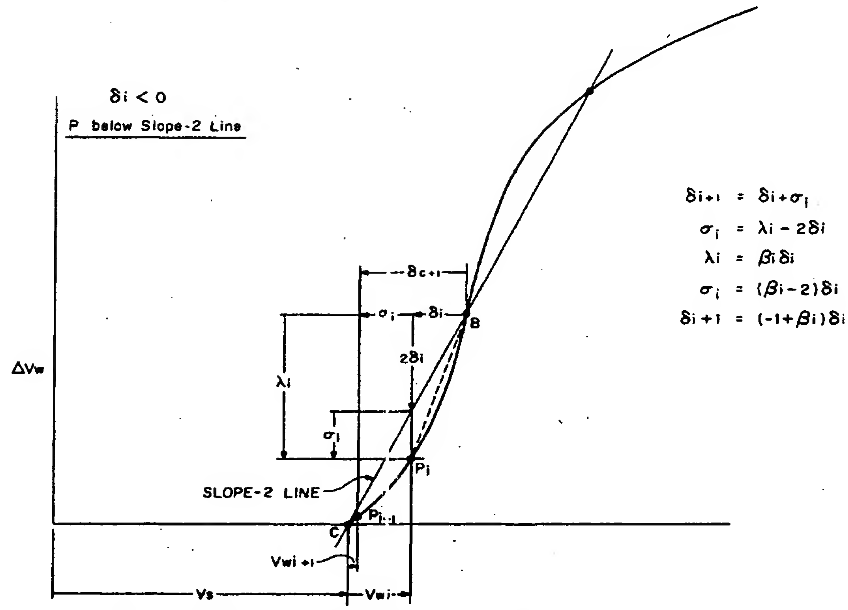

Fig. 4. Movement of operating points in the ac plasma display. Wall voltage less than equilibrium value. Operating point below locus of equilibrium points ($\beta > 2$).

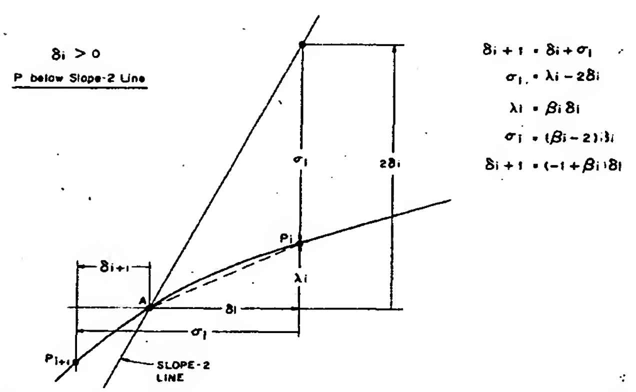

Fig. 5. Movement of operating points in the ac plasma display. Wall voltage greater than equilibrium value. Operating point below locus of equilibrium points ($0 < \beta < 2$).

while B is unstable. If, as in Fig. 2, the voltage transfer curve intersects the horizontal axis to the right of V_s, a wall voltage between V_s and this intersection persists indefinitely, and since $\beta = 0$ in this region, $\delta_{i+1} = -\delta_i$; the operating point oscillates around C.

If the voltage transfer curve intersects the horizontal axis to the left of V_s, the equilibrium point C will be at the lowest intersection of the voltage transfer curve and the slope -2 line. This state is characterized by a sequence of weak discharges.

We have considered the movement of an operating point P with respect to neighboring equilibrium points, but (2)–(6) are valid for all points on the voltage transfer curve with respect to any equilibrium point. For example, if we consider the point P between A and B with respect to the "off" equilibrium point C, we note that the chord from C to P has a slope $\beta > 2$. Equation (6) then states that $|\delta_{i+1}| > |\delta_i|$. Since $\delta_i = V_{wi}$ when C is the reference, the wall voltage V_w increases and P moves away from C and toward A. Similarly, if P is on the right of A, $0 < \beta < 2$, and P will move toward C and, therefore, toward A. In each case β approaches the value 2.

The problem of writing at a discharge site is simply a matter of producing a wall voltage that corresponds to an operating point on the voltage transfer curve that is above the unstable equilibrium point at B. The following sequence of discharges will move the operating point to A where it stabilizes. Usually, it is desirable to reach the neighborhood of A with the initial write discharge. The operating point then reaches A rapidly, in part because it is already close, but also because the value of β is closer to 1 than it is lower on the transfer curve. We note from (6) that if $\beta = 1$, the next discharge will bring the operating point to equilibrium at A. A less intense initial discharge, however, can be useful. Ngo has described a technique in which this kind of write discharge is combined with an erase pulse to flip states at a discharge site [4]. A site that is "on" is not affected by the write pulse but is erased by the erase pulse. A site that is "off" is set by the write pulse to a point close enough to B that the erase pulse causes little or no discharge activity. In the following cycles, the operating point moves upward toward A where it remains as an "on" state. An example of a sequence of discharges approaching equilibrium over many cycles is described by Weber [5].

The problem of erasing is a matter of reducing the wall voltage at an "on" state to a level that corresponds to an operating point below the unstable equilibrium point at

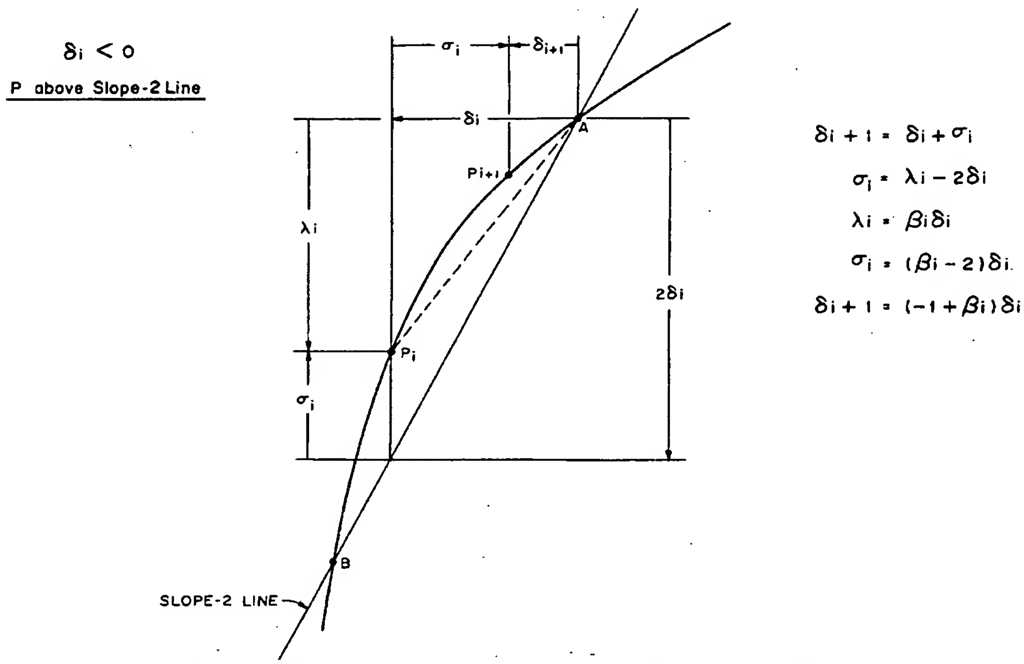

Fig. 6. Movement of operating point in the ac plasma display: Wall voltage less than equilibrium value. Operating point above equilibrium value ($0 < \beta < 2$).

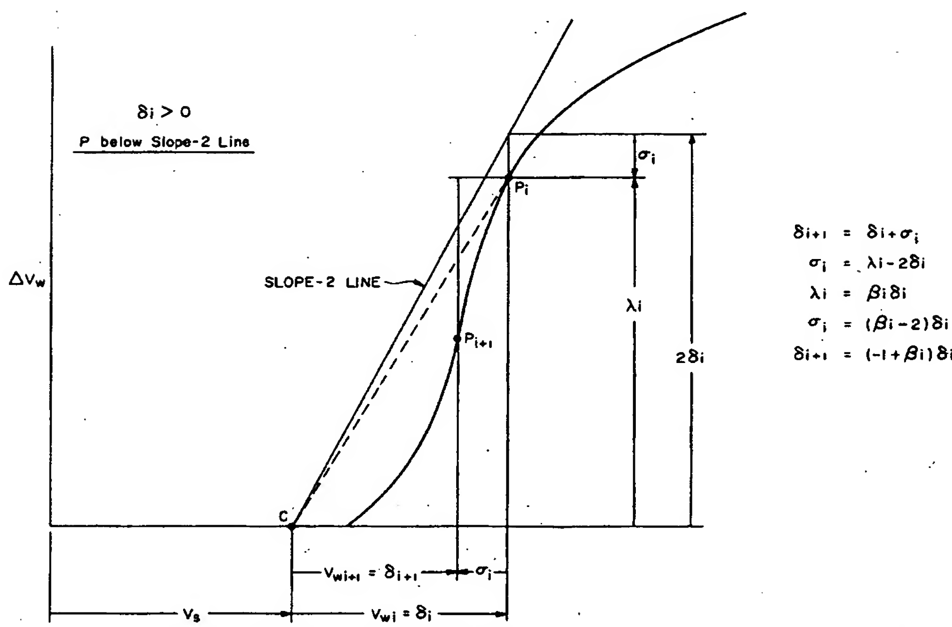

Fig. 7. Movement of operating point in ac plasma display under conditions for which no stable "on" state exists.

B. The following sequence of diminishing discharges moves the operating point to *C*, or if β is zero in this region, to the neighborhood of *C* where it oscillates on either side of *C*.

Within the framework of this model, and for bistable operation, (6) and the curves of Fig. 2 completely define the changes in wall voltage once an initial value has been established. They also imply that, for bistability,

1) the two stable equilibrium points must be separated by an unstable point,

2) between two adjacent equilibrium points, the slope of the voltage transfer curve must exceed 2 in some region, and

3) in the region below the stable "on" point, the transfer curve must rise above the locus of equilibrium points.

If the applied voltage V_s is reduced, the points A and B approach each other, and the bistable region shrinks until at the edge of bistability they merge. For lower values V_s, the transfer curve falls entirely below the locus of equilibrium points, and no value of wall voltage can be preserved in a discharge sequence. Instead, after an initial "write" discharge, succeeding discharges will diminish in both intensity and wall voltage until the voltage at a discharge site is too small to produce a discharge. Fig. 7 illustrates the movement of the operating point P with respect to the "off" equilibrium point in this monostable mode. Through control of the initial discharge Nolan has exploited this property to demonstrate gray scale in the ac plasma display working in a refresh mode. [6] This mode of operation, however, requires a large number of discharges before the sequence extinguishes. This in turn implies that the voltage transfer curve of Fig. 7 should lie below but close to the locus of equilibrium points. An appropriate transfer curve would have a large region for which the slope is, approximately, 2.

It should be pointed out that the shape of the voltage transfer curve depends on the wave form of the exciting voltage [7]–[9]. As a result, the transfer curves are generally different for write, erase, and sustain. This is actually an advantage since it allows independent control of these three functions. The shapes of the transfer curves also depend on the physics of gas discharges that are insulated from the exciting electrodes. The nature of this influence is currently a subject of active investigation [8]–[10].

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